PRODUCTIVITY, PROFITS, AND THE GROWTH OF FINANCING CAPACITY

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Abstract

This essay identifies and addresses the problem of instability that arises in the Lewis (1954) model of growth of financing capacity underlying the development process. In contrast to the unconstrained process in Lewis, it specifies a profit creation process that is subject to constraints on intermediate supply and the supply of savings available for future investment as well as other non-negativity conditions. Under the assumption of profit maximization over an infinite time horizon, the optimal conditions are identified using optimal control theory and the Pontryagin maximum principle. The key finding is that, subject to the method of approaching the optimal technique of production, the optimal level of savings at any time is appropriately specified in terms of three sets of forces: (i) the average product of the knowledge, skills and self-confidence of workers augmented by the elasticities of output and price with respect to that knowledge, skill, and self-confidence; (ii) the unit total cost of production, including the wage cost and the cost of finance; and (iii) the influence of environmental conservation of intermediate capital (natural resources) in lowering the net rate of reinvestment of profits. As the economy approaches full employment, institutional policy interventions to constrain inflation raise the cost of financial capital and complements the rising wage rate caused by labour market forces. The result is convergence of the augmented average product of labour and the average cost of production that reduces the availability of profits to be saved. The growth of intermediate resource use simultaneously causes a falling rate of reinvestment of profits that reinforces the effects of the market forces. The flow of savings ceases to grow, and the Lewis instability problem is avoided accordingly.

Key Words: intermediate inputs, production technique, price, costs, profits, savings, constraints, knowledge, skills, self-confidence, conservation.

1. Introduction

It is now clear from available evidence that self-sustaining increases in the standard of living is achieved through simultaneous and sustained investment to improve three strategic competitive factors that cause productivity growth and enable success in the battle for opportunity in the global marketplace: the capital share of GDP, institutions, and technology (James and Hamilton, 2022). From Lewis (1954), we learned that, subject to a stationary wage rate enabled by the low average product of labour in a relatively large subsistence sector, productivity growth generates the necessary capacity for sustainable financing of these investments by growing profits and savings in the capitalist industrial sectors of the economy. Thus, a competing economy does not have to depend mainly on capital transfers to finance the accumulation of capital assets, institutional development, and technological progress.

To keep the process finite, Lewis envisaged that the creation of savings would be brought to a halt as the economy develops, the subsistence sector is marginalised and its average product of labour rises, and the labour market is pushed to full employment in the capitalist sectors. In the capitalist sector, this would raise the wage rate and cause its convergence to the marginal product of labour. However, Sraffa's (1926, 1930) insightful critique of the standard theory of the firm also implies the possibility of instability, which means indefinite growth of financing capacity, in the Lewis process. If options for technology-driven increasing returns, economies of scope, and learning by doing are continually exploited as a response to changing labour market conditions, then the growth of the marginal product of labour could outpace the growth of the wage rate and lead to indefinite growth of profits and savings that would perpetually lower the cost of capital. With both wages and capital costs considered, any rising wages in the labour market could be sufficiently offset by the falling cost of capital, producing either a stationary or falling unit cost production in combination with a rising marginal product of labour.

Now, for a model to be useful it must yield stable and hence finite solutions. This is a fundamental requirement since solutions that are not stable are useless for policymaking because no measurement based on the model can be exact. The key to solving the Lewis instability problem is to note that his investment financing process ignored the influence of key constraints and boundary conditions on profit creation, especially those related to the availability and use of intermediate inputs and the growth of the stock of capital used by workers to create new value. When these constraints and boundary conditions are incorporated, other related feasible controls can be introduced into the Lewis profit and savings process. These then introduce other options for plausible restrictions that yield a stable (finite) solution, some related to the identification of the optimal technique of production, some to the environmental preservation process, and some to institutional policy interventions to ensure stability in the financial markets through the control of the cost of capital and hence the rate of inflation. This paper contributes by setting out such a constrained profit maximization model in the spirit of Lewis (1954) and using it to investigate the pathway to endogenous growth of savings that nevertheless yields finite financing capacity.

First, the paper specifies the general nature of market-based creation of financing capacity through profitable production using an intermediate capital resource and considering the role of production technique, price, cost, and the size of operations as measured by the scale of the capital stock contributed by capitalists. Second, it represents the role of maximizing behaviour and the related profit-maximizing problem over some institution-time horizon, with associated constraints and boundary conditions. Third, the solution of the constrained profit maximization problem is presented, giving rise to the reference optimal level of savings and the optimal production technique in relation to which competitive strategy can be formulated. Fourth, the elements of competitive strategy to approach the optimum solution are specified, assuming suboptimal initial conditions, and conditions for a finite solution identified that feature institutional intervention to control the rate of inflation. Fifth, the results are summarized.

2. Increasing Financing Capacity through Profits

Let K be the stock of capital assets, Y the rate of output, and N the level of employment of labour. To represent Lewis's (1954) unconstrained model of self-sustaining financing capacity for a developing economy, it is necessary to assume a composite output function $Y(N(K))$ with output depending on the level of employment, which in turn depends on the available stock of capital. To convert intermediate inputs into production based on this function, financial capital is required to cover physical capital costs including profits and depreciation (consumption of fixed capital), wage costs, and taxes. We abstract from taxes and treat profits and depreciation as "profit" generated after meeting wage obligations from the value of output produced. Let $r = \frac{dK}{K}$ be the rate of profit, p the price of output, and w the average wage. As an alternative, the value of output could be measured net of consumption of fixed capital, though the latter is usually quite difficult to measure. Profits were represented as:

1. $rK = pY - wN$

Identification of profit is important since the owners of capital also own the production process, arrange its management, and own its output. They claim depreciation plus profit as recovery of capital plus the reward for this function of ownership. Expansion of employment depends on an increase in the stock of capital available and hence an increase in the capacity to finance this additional stock. This capacity is provided by savings from profits. Since the output function is $Y(N(K))$, then, using the total differential of equation (1), we could write the rate of profit, r, as:

2. $r = \frac{1}{(1+\frac{Kdr}{rdK})}[p\frac{dY}{dN} + Y\frac{dp}{dN} - w - N\frac{dw}{dN}]\frac{dN}{dK}$

Notice that equation (2) implies that the standard full employment equilibrium representation that the value of the marginal product of capital equals the rate of profit is very special and implausible. It only holds under the implausible conditions that: (i) $\frac{Kdr}{rdK} = 0$, so the rate of profit is independent of the rate of accumulation of capital, and (ii) the wage rate is analytically indeterminate since it is also necessary that $w = p\frac{Y}{N}\frac{\frac{Ndp}{pdN}}{(1+\frac{Ndw}{wdN})}$ and under full employment $\frac{\frac{Ndp}{pdN}}{(1+\frac{Ndw}{wdN})} = \frac{\infty}{\infty}$.

Now, make the Lewis surplus labour assumption that $\frac{dw}{dN} = 0$, and hold price invariant to employment growth, so $\frac{dp}{dN} = 0$. Treat saving as the component of the value of output not used for final consumption and assume that all wages are consumed. Further, write the rate of saving out of profits as $s_p = \frac{SY}{YdY}\frac{dY}{dK} = \frac{s_y}{g_y}\frac{dY}{dK}$, where S is the level of savings and $\frac{s_y}{g_y}$ is the ratio of the rate of savings out of income to the rate of growth of income. Then, the level of savings of the economy in any period, S, is given by:

$$3. \quad s_p rK = S = \frac{s_p N}{(1+\frac{Kdr}{rdK})}\left(p\frac{dY}{dN} - w\right)\frac{KdN}{NdK}$$

Equation (3) implies that under surplus labour conditions, with w stationary, if investment grows the marginal product of labour continuously, then the savings rate will grow indefinitely. To keep the process finite, Lewis envisaged that S would cease to grow as $\frac{dN}{dK}$ increases in the capitalist sector, the subsistence sector is marginalised and its average product of labour rises, and the wage labour market is pushed to full employment. In the capitalist sector, this process would raise w and cause $w \to p\frac{dY}{dN}$, causing $r \to 0$ and hence $S \to 0$. The Sraffa (1926, 1930) argument implies the possibility of instability, that is indefinite growth of financing capacity, in equation (3). As the economy approaches full employment, it is highly plausible that $\frac{dK}{K}$ grows faster than $\frac{dN}{N}$ implying rapid changes of technology. Then, if options for technology-driven increasing returns, economies of scope, and learning by doing are continually exploited as a response to changing labour market conditions, the growth of $\frac{dY}{dN}$ could outpace the growth of w. In that case, $\frac{dY}{dN} > w$ in perpetuity and the level of savings is unstable.

One way to address this problem of unstable growth of financing capacity is to admit the cost of capital and regulatory processes into the development process. This requires an alternative model of profit creation that: (i) explicitly represents the technology of production for the market as a process of work to transform intermediate inputs; (ii) is constrained by forces that reflect the cost of financial capital and the role of time discounting as a targeted rate of growth of financing capacity; and (iii) accounts for consumption of the fixed capital installed to enable production.

The market has long been seen as omnipresent in modern economies and economists have long sought to develop methods for studying how it is coordinated by the process of competition (Smith, 1776; Polanyi, 1944; Polanyi, et al, 1954). Importantly, the market provides avenues for acquiring and accumulating financing capacity through production, which is essentially a problem-solving process that creates new output by use of intermediate inputs. In a market, trades of some quantity of property used as intermediate input, x, involve the interaction of buyers and sellers, who agree on a strike price, $v(x)$, plus any associated conditions under which x is exchanged. In the production process described by the composite function $h(v(x))$, $v(x)$ is fully embedded in the resulting output. The market price (p) of the new output is the same as the price of $h(v(x))$ so one can represent the value of the new output as $ph(x) = ph(v(x)) - v(x),$ where $h(x)$ is the volume of the net output. In basic accounting terms, $ph(x)$ is the value added by use of $v(x)$ in production. Notice that if intermediates are free, then $h(x) = h(v(x))$. In general, p and thus $ph(x)$ is governed by the intrinsic quality and reliability of x as a problem-solver in production.

Production is work with x done by workers and managers with knowledge, skills, and self-confidence ($\widetilde{N}$) working under specific institutional arrangements with complementary produced assets, K, which are assembled by capitalists who own the new output. In producing for the market, capitalists normally assemble capital, K, by use of financing capacity or savings (S) for investment at a cost c_f. The cost of production of $h(x)$ can be represented summarily as $c(h(x))$, which is the sum of the cost of access to worker effort over some agreed chronological time plus the cost of the services obtained from the assembled capital in that time. In practice, one can write the unit cost of using x in production as $c(x) = \frac{c(h(x))}{x}$. Traders can pursue increased value generated from continuous use of x to solve problems and generate sales of the new output, $ph(x)$, net of the cost of using x, $c(x)x$.

Even though working arrangements are contracted for a specific chronological time, the volume of work and new output in that time is variable mainly because of the influence of institutions. All production-related variables are functions of institution-time (t), in the sense of being subject to the institutional arrangements that coordinate market participation and determine the speed of movement and the extent of work effort. In a given chronological time, more effective institutions lead to more work effort and better output than is possible with less effective ones. In particular, t represents the continued sequence in which economic events occur in an irreversible succession, and hence the duration of events or the intervals between them. This means it also allows representation of rates of change and acceleration of change of the events. In society, the meaning of t differs for different institutional frameworks, because the institutions govern the sequence and duration of events. In other words, t only has meaning relative to a particular set of governing institutions. To describe observations of an economic event, a timescale for the event to occur associated with its governing institutions must also be noted. It is in this substantive sense that t refers to institution-time. The applicable institutional arrangement and the level of institutional coordination govern the level of aggregation represented by the unit being modelled. This means that units can be analysed at the level of the firm, the industry, or the economy depending on the mechanisms of institutional coordination represented by t.

The net financial gain (or profit) generated by using x in the problem-solving process can be represented generally as:

4. $\pi(t) = ph(x(t)) - c(x)x(t)$

Clearly, π can be reformulated in other ways appropriate to the type of analysis being conducted in the light of the facts of the case. At the level of the economy, from the perspective of financing capacity, $\pi + c(x)x(t)$ can be set up to measure the total financial capital needed to transform intermediate inputs into output, and thus set up as the sum of the wages, depreciated machinery and profits (including surpluses of government enterprises), plus taxes. In this analysis, we abstract from taxes for convenience.

The treatment of π as a function of the underlying institutional framework reflects the nature of competition among market participants, which has been considered the primary coordinator of market operations since Smith (1776). In society, nothing, so to speak, is independent of institution-time. Institution-time can change slowly or rapidly, giving more than chronological meaning to the mathematical notions of Δt and dt as the change of institutions in chronological time.

Competition is fought with a spectrum of methods that revolve around production technique which is shaped and constrained by the rules and regulations of law, as well as by the path of institutions such as the education and training systems, the framework of international engagement and collaboration, and the monetary and financial institutions. Production technique directly determines productivity and unit costs, and hence financial gain or profits.

In the process of competition for profits, there is no global coordinating government and no established guidelines that create a multilateral and multidisciplinary mechanism for participatory control, management, and evaluation of the exchange of the process, including with respect to its driving technologies. There is no international fund for the financing of technology sharing that bypasses the principles of intellectual property rights. Much of knowledge is private and rivalrous rather than universal non-rivalrous public property, making accessibility a substantial challenge. In that context, private intellectual property and private capital are critical elements in the developing technologies that yield novel winning solutions and drive the battle of competition. There is no overarching agreement on the principle that those market participants who come up with winning technologies have a responsibility to share the technology with those who do not, by building research and knowledge centers to aid them in the process of adoption, adaptation, and creation of their own winning technologies and innovations in the process of competition. As a result, in the competitive process, there are successes and failures, winners and losers.

However, if left unregulated, competitors will also deploy direct competitive initiatives, especially techniques, designed to gain access to (or avoid losing) markets and profits while hurting the chances of others to do so. These can also take the form of advertisements, political, legal and social initiatives, espionage and even technical sabotage or militaristic conquest. Similarly, if left unsupported by the institutional framework, production techniques can stagnate and result in loss of markets and profits. When permitted, supported, and successful, competitive technical initiatives determine the levels of surplus-value created and hence the capacity to fund continued development of competitive strategy. When institutional arrangements such as the rule of law constrain the deployment of some of these direct competitive initiatives, production technique is significantly affected, so in what follows below π reflects the underlying forces of institution-time.

It is assumed that the individuals, firms, or other entities participating in the market are all interested in growing the produced π over time at some rate g. Assuming continuous growth under the influence of institution-time, define:

5. $\pi(t+\Delta t) = \pi(t) + g\pi(t)\Delta t$

Here, g, the rate of growth of $\pi(t)$, is also substantively its rate of reinvestment. It should be clear from equation (4) that π grows through growth and transformation of $h(x(t))$ faster than the growth of $c(x)$ as institutions develop. The economics concerns the underlying contextual details on the way factor markets work and the methods of accumulating capital and increasing knowledge, skills, and self-confidence to solve the problems of society. Setting $\pi\Delta t = \Delta\pi$ gives:

6. $\Delta\pi = g\pi(t)\Delta t$

Or, taking limits:

7. $\frac{d\pi}{dt} = g\pi(t)$

Separating variables, solving for π and setting $\pi(0) = \pi_0$, gives:

8. $\pi(t) = \pi_0 e^{gt}$

If we set $\pi(t) = 1$ for a unit of profit, then the value in period 0, the present value, of one unit of profit to be gained t periods hence when profit grows continuously at the rate g, is:

9. $\pi_0 = e^{-gt}$

Now, let $\pi(t)$ be the total amount to be earned in period t. Then, the present value of $\pi(t)$ is:

10. $\pi_0 = \pi(t)e^{-gt}$

Summing over t periods and letting $t \to \infty$ gives the present value over all periods as:

11. $PV = \int_0^\infty e^{-gt}\pi(t)dt$

Instead of continuous growth, it might be appropriate to assume discrete growth. Accordingly, it is useful to observe here that discrete growth is the special case of equation (8) in which $g = \ln(1 + c_f)$, where c_f is the cost of capital; so $e^{gt} = (1 + c_f)^t$. In that case equation (9) translates to:

12. $\pi_0 = \frac{1}{(1+c_f)^t}$

Further, equation (10) becomes:

13. $\pi_0 = \frac{\pi(t)}{(1+c_f)^t}$

And, equation (11) becomes:

14. $PV = \sum_{t=1}^{n} \frac{\pi(t)}{(1+c_f)^t}$

The choice to use either equation (11) or (14) depends on the facts of the case and the issues being analysed. The rest of this analysis makes the continuity assumption and therefore uses equation (11).

3. Constrained Profitmaking

If x is treated as the property of the analytical unit with use-value (or utility) in problem solving, an approach that explains maximizing behaviour and provides foundations for the analysis of competition can be developed based on two key assumptions. One assumption is that, in a capitalist market process, profit is based on the acquisition and employment of property in combination with the knowledge, skills, and self-confidence of workers hired for production. The other assumption is that profit creation is constrained by the availability of x, the technique of use, and the capital resources accumulated to use it. The market constraints are expressed using differential or difference equations. When framing these assumptions, the influence of competition and competitive strategy must be explicitly considered. Competition is fought with price, production technique, and productivity, each of which is constrained by the applicable institutions, including the rules and regulations of law.

Equation (4) represents the production process. The general logic is that all demand for x is derived demand, which is to say demand for x as an input in production of other products or services. Thus, demand for x is derived from the final demand for the products or services it can help to produce in combination with other inputs, which are generally the capital assets contributed by capitalists, K, and those contributed by workers, $\widetilde{N}$. The principle is quite general, in the sense that it includes the cases of use in personal production for own consumption as well as commercial production for sale and profit.

The financial gain defined as profit is created when the owner of intermediate property x and K hires the knowledge, skills, self-confidence, and labour-power of workers ($\widetilde{N}$) to use them to create new value at some cost. $\widetilde{N}$ is applied through a technique and augmented by the stock of capital. The effort to extract the use-value in x drives demand, whether direct or derived, to use x, and leads to its depletion or depreciation, obsolescence and discard because of relative technological backwardness and economic inefficiency that leads to high costs. In the case of direct demand, the new value is consumed during the creation process. In the case of derived demand, the new value takes the form of capitalist property (a good or service) to be sold in the market. The technique employed is most conveniently defined as $\phi = \frac{\widetilde{N}}{K}$, with $\widetilde{N} = E_n N$, for E_n the average level of knowledge, skills, self-confidence, and labour power of workers and N the number of workers. It follows that skilled work effort is $\widetilde{N} = E_n N = \phi K$. In turn, this implies that $0 \leq \phi \leq 1$.

Production technique is a combination of the totality of current scientific and experimental knowledge, skills, methods, and processes in the possession of the workers, as well as that embodied in the machines, infrastructure and other assets assembled by capitalists. The definition makes sense of the fact that much of technology tends to be embedded in capital (machines and other physical plant and infrastructure) assets that can be applied and operated without an individual or firm having detailed knowledge of their inner workings. It follows that a change in ϕ represents changes in the inputs used and products created, especially with respect to their problem-solving capacity and hence their reliability and quality.

In a capitalist economy, it is necessary to treat K as the resources contributed and controlled by capitalists, while $\widetilde{N}$ is owned by workers who work under the control of the capitalists. However, other sociological constructs are admissible. It is appropriate to treat x as an intermediate capital input into productive consumption. The level of output created with it can then be treated as the derived demand for x and written as the product of work effort and x. Thus, we can write:

15. $h(x,t) = \phi K x^{\beta}$

Notice that ϕK is a scale factor for x and that the larger is K, the higher is the level of profits achieved in equation (4).

To account for the cost of all inputs, let c_v be the variable cost per unit of effort, essentially covering the wages and salaries of managers and workers who bring knowledge, skills and self-confidence to the work effort. Also, let c_f be the (weighted average) cost (per dollar) of financial savings (S) used to invest in building up the stock of physical capital, including training workers and managers and providing working capital to operate it. The total cost of productive effort is then given by $c_v\phi K + c_f S$. Then, if p is the price of the benefits or new value form (property) created by use of x, the profit created is given by:

16. $\pi = p\phi K x^{\beta} - c_v\phi K - c_f S$

Using (16) in (11) gives:

17. $PV = \int_0^{\infty} e^{-gt}(p\phi K x^{\beta} - c_v\phi K - c_f S)dt$

In equations (16) and (17), c_v and c_f will tend to reflect the factor market supply created by the availability of capital to employ the supply of workers in the economy with some average level of knowledge, skills, and self-confidence E_n.

The constraint on surplus creation posed by x is expressed by a differential equation of net supply that uses $F(x)$ and $h(x,t)$ to represent the net growth of economic opportunity. In particular, the net supply of economic opportunity to create net value based on any commodity x in the market is represented by a simple state equation that treats $F(x)$ as the rate of growth of the supply of x and $h(x,t)$ as the rate of user demand for x, or its rate of depletion, in the process of problem-solving and productive consumption. The resulting DE of net supply of x is the state equation:

18. $\frac{dx}{dt} = F(x) - h(x,t), x(0) = x_0$

That is, the net supply of economic opportunity to create net worth using x is the gap between the developing market supply and market demand. Viewed in isolation, equation (18) is an initial-value problem that is known to be well-posed in some domain $\mathcal{D}$ when there exists a unique solution that also varies continuously with the usual constant of integration. The mathematician only cares about whether the composite function $F - h$ can be integrated, and it sometimes cannot be without reliance on numerical methods. For the economist, a market cannot exist if either $F(x) = 0$ or $h(x) = 0$. If $h(x) = 0$, suppliers will not deliver any x and $F(x) = 0$. Similarly, if $F(x) = 0$, demand cannot be realized, so $h(x) = 0$. In the DE in equation (18), it is evident that when $h(x,t) > F(x)$, excess demand exists, $\frac{dx}{dt} < 0$ and net economic opportunity declines. When $h(x,t) < F(x)$, there is excess supply, $\frac{dx}{dt} > 0$ and net economic opportunity grows. When $h(x,t) = F(x)$, $\frac{dx}{dt} = 0$. Points like these are critical points in the model.

Just as there is a market for x, so too there is one for K, which is the pool of properties owned by capitalists that are produced and accumulated as assets used in the creation of new value with x. It is generally agreed that K is built up through savings (S) from previous profits that are used to finance investment, while provisions for replacement of the existing stock of capital grow with use-related demand and depletion (depreciation), obsolescence and efficiency at some rate γ. Thus, the constraint on equation (17) posed by the net supply of capitalist financing capacity can be written as the state equation:

19. $\frac{dK}{dt} = S - \gamma K, K(0) = K_0$

Here, it is noted that if $\gamma K > S$, or if $S = 0$, then $\frac{dK}{dt} < 0$ and the value of the stock of capital is depreciating continuously. An economic unit cannot exist under such conditions for very long. This points to the fact that while equations (18) and (19) are the main supply constraints on the creation of surplus-value in the surplus equation (17), there are other associated constraints, such as $S(t) \geq 0$ in equation (19), and $0 \leq \phi(t) \leq 1$ in equation (18). Even more interesting for Caribbean economies is when profit-generating capacity causes S to be low relative to γK, then $\frac{dK}{dt} \to 0$. In that case, with D an instance of the differential operator, $S = DK = \gamma K$ is a critical point. Then, the stock of capital is given in institution-time by $K(t) = K_0 e^{\gamma t}$. Thus, if institutions change slowly and institution-time changes on a long time scale, then $t \to 0$ and the stock of capital remains tied closely to the initial stock. This essentially makes the capital stock an independent force determining production in equation (15).

It is then assumed that the interests of the unit in the market is to maximize PV, the present value of profit-based financing capacity, which depends on $x(t)$, $h(x,t)$ and S in the state equations. If we conveniently set $\beta = 1$ in equation (15), this intent can be expressed in complete constrained form as:

20. $\max PV = \int_0^\infty e^{-gt}[(px - c_v)\phi K - c_f S]dt$

Subject to

21. $\frac{dx}{dt} = F(x) - \phi K x, x(0) = x_0$
22. $\frac{dK}{dt} = S - \gamma K, K(0) = K_0$
23. $0 \leq \phi(t) \leq 1$
24. $S(t) \geq 0$

Several observations are relevant here. First, equation (20) involves maximization of an improper integral. So, $e^{-gt} \to 0$ rapidly as $t \to \infty$ or as t becomes large, and there is a discontinuity at ∞ at which the outcome breaks down. Second, if $g = 0$, maximization effectively occurs without time discounting, implying that use of the resources brings the same permanent benefits in profits over time. On the other hand, if $g = \infty$, the associated discontinuity makes outcomes indeterminate. Third, and important for conventional methodology, equation (20) indicates that in the market $PV = f(g, p, c_v, c_f)$, which implies that g is determined by an inverse function such that $g = g(PV(p, c_v, c_f))$. This then implies that g could not be used in a market price-determination model in which the prices determined are p, c_v, c_f.

4. Characterizing the Optimal Choices

To characterize necessary optimizing competitive behaviour (trends and requirements) by individuals and firms seeking to create surplus-value, it is first necessary to solve the system in equations (20) to (24). For this, we can appeal to a powerful optimization method that allows examination of multi-dimensional and nonlinear specifications in the equations. This method is provided by optimal control theory and Pontryagin's famous maximum principle (Pontryagin, *et al*, 1962). The maximum principle itself is a set of necessary conditions for optimality that can deal with nonlinearity and inequality constraints.

The maximum principle is most conveniently formulated in terms of the Hamiltonian:

25. $$\mathcal{H} = e^{-gt}\left[(px - c_v)\phi K - c_f S\right] + \lambda(t)[F(x) - \phi K x] + \mu(t)[S - \gamma K]$$
$$= [e^{-gt}(px - c_v)K - \lambda K x]\phi(t) + S(t)\left[\mu - e^{-gt}c_f\right] + \lambda F(x) - \mu\gamma K$$

where $\lambda(t)$ and $\mu(t)$ are Lagrangian multipliers or adjoint variables and $\phi(t)$ and $S(t)$ are control variables, the variables to be manipulated when devising and implementing competitive strategy. The concept of a control variable is logically consistent with treatment of t as the general underlying independent variable, institution-time, driving all other forces in the profit-creation process.

In the first line of equation (25), the first term $e^{-gt}\left[(px - c_v)\phi K - c_f S\right]$ is the maximand of equation (20), which is a function of the control variables. It is the value-flow of discounted financial returns from using x and K into the total accumulated returns in the market defined by $\mathcal{H}$. In substance, the maximand is the competitive financial performance of the competitor in the market at t. Importantly, the competitive performance and associated total accumulated returns can be defined at any level of aggregation, individual, firm, industry or economy, as long as subscripts are chosen carefully and institutions are appropriately identified.

The adjoint variable $\lambda(t)$ measures the rate of growth of the present value of surplus (the maximand) due to growth of x at t, while $\mu(t)$ measures the rate of growth of the present value of surplus (profits) due to growth of K at t. These are what economists call *marginal values*. If $\lambda(t)$ or $\mu(t)$ is negative, it represents the Keynesian marginal user cost or the rate of loss of the present value of surplus. This interpretation assumes that change is occurring along the optimal trajectories of the assets, which we label x_{opt} and K_{opt}. Further, $\lambda(t)$ and $\mu(t)$ are also called the shadow prices of x and K respectively, to be distinguished from the sale prices. These are imputed values that are based on resource productivity and productivity growth over time and might be best interpreted as the prices it makes sense to pay now for properties that contribute in the identified ways to growth of the total present value of all assets. Since the maximand is a measure of competitive performance in the market, $\lambda(t)$ and $\mu(t)$ are measures of the rate of improvement in competitive performance due to use of x and K respectively.

The other two terms in line 1 of equation (25) are the Lagrangian constraints on the maximand, which together are also functions of control variables. The first of these, $\lambda(t)[F(x) - \phi Kx]$, is the present value of the flow of the input x into the total accumulated returns defined by $\mathcal{H}$. Importantly, $F(x) - \phi Kx$ is the net flow (rate of increase) of the asset x, so it is not expressed in present value units. The second Lagrangian constraint, $\mu(t)[S - \gamma K]$, is the present value of the flow of financial capital into the total accumulated returns defined by $\mathcal{H}$, again with $[S - \gamma K]$ not expressed in present-value units. Thus, overall, $\mathcal{H}$ is the rate of increase of the present value of the assets used by the competitor in the market for x. The second line of the equation indicates that the problem is linear in the control variables $\phi(t)$ and $I(t)$, with two switching functions which are their coefficients. The optimal solution that maximizes $\mathcal{H}$ is some combination of $\phi(t)$ and $I(t)$, as well as of x and K, and the competitive strategies implied in achieving them.

The maximum principle asserts the necessity that values of $\phi(t)$ and $S(t)$ exist which at least maximize the rate of increase in the total value of the assets, $\mathcal{H}$, for all t, given λ and μ. That is:

$$26.\ \mathcal{H}(x(t), K(t), t, u(t), \lambda(t), \mu(t) \geq \max_{u} \mathcal{H}(x(t), K(t), t, u(t), \lambda(t), \mu(t)$$

where $u = \{\phi, S\}$. Optimal choice of $\phi(t)$ depends on λ and optimal choice of $S(t)$ depends on μ, thus the maximum principle reduces the optimal problem to determination of λ and μ. However, determination of λ and μ by simple analytics is a very difficult problem, which is often not solvable. The maximum principle addresses this problem by also asserting that if $\phi(t)$ exists such that $x(t)$ is a response, then there exists $\lambda(t)$ such that for $0 \leq t \leq T$:

$$27.\ \frac{d\lambda}{dt} = -\frac{\partial \mathcal{H}}{\partial x} = -\frac{\partial G}{\partial x} - \lambda(t)\frac{\partial M}{\partial x}$$

where $G = e^{-gt}\left[(px - c_v)\phi K - c_f S\right]$ and $M = F(x) - \phi Kx$. Also, if $S(t)$ exists such that $K(t)$ is a response, then there exists $\mu(t)$ such that for $0 \leq t \leq T$:

$$28.\ \frac{d\mu}{dt} = -\frac{\partial \mathcal{H}}{\partial K} = -\frac{\partial G}{\partial K} - \mu(t)\frac{\partial J}{\partial K}$$

where $J = S - \gamma K$.

In equation (27), since $\lambda(t)$ is the rate of growth of the total present value of all assets due to growth of x at t, $-\lambda(t)$ is the rate of loss of present value (marginal user cost) or value depreciation. Thus, $\frac{d\lambda}{dt}$ indicates that the rate of acceleration of present value equals the sum of the rate of deceleration of returns and the rate of deceleration of investment in (rate of depletion of) x. In equation (28), $\mu(t)$ is the rate of growth of the present value of total assets due to growth of K at t, $-\mu(t)$ is the rate of loss of present value associated with growth of K. Thus, $\frac{d\mu}{dt}$ indicates that the rate of acceleration of total present value due to growth of K equals the sum of the rate of deceleration of returns and the rate of deceleration of investment in K. Overall, equations (27) and (28) indicate that along the optimal path, the rate of depreciation of total present value is sum of the rate of growth of financial returns and the rate of growth of investments, which is to say the overall rate of accumulation of assets.

Equations (26) to (28) are necessary conditions to be satisfied by the optimal values of $\phi(t)$ and $S(t)$. Equations (27) and (28) are conditions for exactness. Furthermore, suppose that for all possible values of $u = \{\phi, S\}$ no value satisfies equation (26). Then, $\max_{u} \mathcal{H}(x(t), K(t), t, u(t), \lambda(t), \mu(t))$ is necessarily found where the derivative of $\mathcal{H}$ with respect to u is zero. That is, where:

29. $\frac{d\mathcal{H}}{du} = 0$

Counting equations, we now have 6 equations to determine 6 unknowns: $x(t)$, $K(t)$, $\lambda(t)$, $\mu(t)$, $\phi(t)$ and $S(t)$. The 6 equations are assembled here as:

30. $\frac{dx}{dt} = F(x) - \phi K x, 0 \leq t \leq T$
31. $\frac{d\lambda}{dt} = -\frac{\partial \mathcal{H}}{\partial x} = -\frac{\partial G}{\partial x} - \lambda(t)\frac{\partial M}{\partial x}$
32. $\mathcal{H}(x(t), K(t), t, u(t), \lambda(t), \mu(t) = \max_{\phi} \mathcal{H}(x(t), K(t), t, u(t), \lambda(t), \mu(t)$
33. $\frac{dK}{dt} = S - \gamma K$
34. $\frac{d\mu}{dt} = -\frac{\partial \mathcal{H}}{\partial K} = -\frac{\partial G}{\partial K} - \mu(t)\frac{\partial J}{\partial K}$
35. $\mathcal{H}(x(t), K(t), t, u(t), \lambda(t), \mu(t) = \max_{I} \mathcal{H}(x(t), K(t), t, u(t), \lambda(t), \mu(t)$

Equations (30) and (33) are DE(1). Their solutions require initial values or boundary conditions, so we also have:

36. $x(0) = x_0$
37. $x(T) = x_T$
38. $K(0) = K_0$
39. $K(T) = K_T$

The initial conditions in (36) and (38) can be met in a variety of ways, including through foreign investment as proposed by Lewis (1950; 1954) or through inclusive innovative development credit as Lewis (1954) implied. As pointed out by Best and Levitt (1969) and Best (1980), initialisation by foreign investment leads to accumulation of financing capacity on the foreign account. Equations (30) to (39) are the right number of conditions for a unique solution, assuming no linear dependence. However, a unique solution will only exist if optimal values of $\phi(t)$ and $S(t)$ exist that satisfy equations (32) and (35). In general, analytic (closed form) solutions may be difficult to find and numerical solutions may be needed.

In pursuing the solution to the problem, the key questions that arise are whether $\phi(t)$ and $S(t)$ are feasible controls or elements of competitive strategy. Then, if feasible, what are their optimal values and how are they approached? In applying the maximum principle to answer these questions, it is admissible that the controls $\phi(t)$ and $S(t)$ might be *singular* in the sense that competitive activity is pursued under conditions in which their coefficients vanish. In this regard, there are four possible cases discussed in detail by Clark (1976; 2010): (i) $\phi(t)$ is singular; (ii) $\phi(t)$ and $S(t)$ are both singular; (iii) $S(t)$ is singular and $\phi(t) = 0$; and (iv) $S(t)$ is singular and $\phi(t) = 1$. The solution is pursued in the context of these cases.

Case 1: $\phi(t)$ Singular

Since $\phi(t)$ is assumed to be singular, its coefficient is set to zero. That is:

40. $e^{-gt}(px - c_v)K - \lambda Kx = 0$

Or,

41. $\lambda(t) = e^{-gt}\left(p - \frac{c_v}{x}\right)$

Thus,

42. $\frac{d\lambda}{dt} = -ge^{-gt}\left(p - \frac{c_v}{x}\right) + e^{-gt}\frac{c_v}{x^2}\frac{dx}{dt}$

Then, using equation (30) for $\frac{dx}{dt}$ gives:

43. $\frac{d\lambda}{dt} = e^{-gt}\left[-\delta\left(p - \frac{c_v}{x}\right) + \frac{c_v}{x^2}(F(x) - \phi Kx)\right]$

Applying the adjoint equation (31) gives:

44. $\frac{d\lambda}{dt} = -e^{-gt}p\phi K + \lambda\phi K - \lambda F'(x)$

Using λ from equation (41) gives:

45. $\frac{d\lambda}{dt} = e^{-gt}\left[-p\phi K + \left(p - \frac{c_v}{x}\right)\phi K - \left(p - \frac{c_v}{x}\right)F'(x)\right]$

Equating (43) and (45) and cancelling terms gives:

46. $\frac{F(x)}{x} = \frac{px - c_v}{c_v}(g - F'(x))$

In general, from equation (30), the LHS of equation (46) is the relative rate of growth of the gross stock of intermediate capital used in surplus production. This is normally the sum of the net rate of growth of intermediate capital and the level of knowledge, skills and self-confidence of workers deployed in production. The RHS is the product of the net cash flows per dollar of cost and the rate of accumulation of profits net of the rate of acceleration of supply of x. That is, it is the net reinvestment of profits per dollar of cost. Equation (46) therefore defines an equilibrium as occurring when the relative rate of growth of the supply of intermediate capital ($\frac{F(x)}{x}$) equals the net reinvestment of cash flows per dollar of cost ($\frac{px - c_v}{c_v}(g - F'(x))$). If labour market conditions keep down c_v, then the effect on (46) is to increase the net cash flows per dollar of cost and hence increase the relative rate of growth of intermediate capital ($\frac{F(x)}{x}$). Similarly, forces that increase the rate of reinvestment g will also increase the net rate of reinvestment $(g - F'(x))$ and therefore increase $\frac{F(x)}{x}$. It is less plausible, but not impossible that, for given c_v and p, $F'(x)$ can be autonomously reduced as production of x

grows, causing the net reinvestment (the rate of accumulation of profits) to increase and with that the relative rate of growth of intermediate capital. Lewis (1954) suggested that wage suppression conditions exist when the stock of capital and the knowledge, skills and self-confidence of workers is too small to fully employ all available workers. Lewis (1954) also suggested that under such conditions, from a causal standpoint, institutional development financing to increase the production of capital will expand options to increase investment and hence the stock of capital while raising the productivity of labour.

However, note that equation (46) has no term in $\frac{dx}{dt}$ and is *singular* in the sense that its solution cannot be derived from the equation by any choice of the arbitrary constants of integration in its integral. To determine if there is an equilibrium solution, one cannot appeal to existence and uniqueness theorems. But it will generally be possible to find an optimal solution $x = x^*_{var}$ which is a constant associated with the variable cost c_v that satisfies the equation. Then, from the state equation in (30), the optimal amount of knowledge, skills and self-confidence employed in production is $(\phi K)_{var} = \frac{F(x^*_{var})}{x^*_{var}}$. Further, from the state equation (33), it would hold that $S_{var} = \frac{\gamma F(x^*_{var})}{\phi x^*_{var}}$, which by equation (46) would make the savings rate at any t be conditional on c_v, akin to the Lewis process.

Case 2: $\phi(t)$ and $S(t)$ both Singular

Now, we assume that the coefficients of $\phi(t)$ and $S(t)$ are both zero. Then, in addition to Case 1, we have:

47. $\mu - e^{-gt}c_f = 0$

Or,

48. $\mu = e^{-gt}c_f$

Therefore, institutional progress yields

49. $\frac{d\mu}{dt} = -ge^{-gt}c_f$

Next, by adjoint equation (34):

50. $\frac{d\mu}{dt} = [e^{-gt}(px - c_v) - \lambda x]\phi + \mu\gamma$

Using λ from equation (41) in equation (50) gives:

51. $\frac{d\mu}{dt} = \mu\gamma$

Equating (49) and (51) gives:

52. $\mu\gamma = -ge^{-gt}c_f$

Since $g, k_f, \gamma > 0$, equation (52) is a logical contradiction. It follows that $\phi(t)$ and $S(t)$ cannot be both singular. This implies that if $\phi(t)$ is singular, as in *Case 1*, then $S(t) = 0$. However, from the constraint in equation (44), $S(t) = 0$ implies that $\frac{dK}{dt} = -\gamma K$, indicating that the physical capital stock is depreciating continuously. Such a solution cannot prevail for very long if any firm, industry, or economy is to survive in the pursuit of surplus-value.

Case 3: $S(t)$ Singular and $\phi(t) = 0$

By the same logic as in Case 2, if $S(t)$ is singular and $\phi(t) = 0$, equation (51) also results from equation (50) and the logical contradiction in equation (52) is reproduced. Furthermore, from the constraint in equation (30), $\frac{dx}{dt} = F(x)$ and x is being accumulated even if there is no derived demand for it and hence no cash flows to support its use. That situation also cannot persist for very long if any firm, industry, or economy is acting rationally and is to remain viable.

Case 4: $S(t)$ Singular and $\phi(t) = 1$

From Case 2, the singularity of $S(t)$ implies equation (49) and the adjoint equation implies equation (50). Equating these equations, substituting for μ from equation (48), and setting $\phi = 1$ gives:

53. $\lambda(t) = e^{-gt}[p - \frac{c_v+(g+\gamma)c_f}{x}]$

Here, $c_v + (g + \gamma)c_f$ is the sum of the cost per unit of knowledge and skills embedded in workers plus the cost of finance (savings) used to invest in new capital owned by the capitalists and run the operations. So,

54. $\frac{d\lambda}{dt} = -ge^{-gt}\left[p - \frac{c_v+(g+\gamma)c_f}{x}\right] + e^{-gt}[\frac{c_v+(g+\gamma)c_f}{x^2}\frac{dx}{dt}]$

Again, using equation (30) for $\frac{dx}{dt}$ in equation (54) gives:

55. $\frac{d\lambda}{dt} = -ge^{-gt}\left[p - \frac{c_v+(g+\gamma)c_f}{x}\right] + e^{-\delta t}[\frac{c_v+(g+\gamma)c_f}{x^2}(F(x) - Kx)]$

Applying equation (44) obtained from the adjoint equation in (31), and using $\lambda(t)$ from equation (53) gives:

56. $\frac{d\lambda}{dt} = -e^{-gt}pK + e^{-gt}[p - \frac{c_v+(g+\gamma)c_f}{x}]K - e^{-gt}[p - \frac{c_v+(g+\gamma)c_f}{x}]F'(x)$

Then, equating (55) and (56), cancelling terms, reorganising and setting the total cost of production as $c_{tot} = c_v + (g + \gamma)c_f$, gives:

57. $\frac{F(x)}{x} = \frac{px-c_{tot}}{c_{tot}}(g - F'(x))$

Equation (57) is the singular solution for the equilibrium value of $x = x^*_{tot}$ associated with c_{tot}. Here again, the LHS of equation (57) is the relative rate of growth of the supply of x. The RHS is the net value of reinvested profits per dollar of cost. Equation (57) therefore defines an equilibrium as existing when the relative rate of growth of the supply of x equals the net value of reinvested net cash flows per dollar of cost generated by the assets and technology engaged in the use of x in production. Here too, if factor market conditions keep down c_v and c_f and therefore c_{tot}, or if the net rate of reinvestment $(g - F'(x))$ increases, the effect is to increase the relative rate of growth of intermediate capital, x. The Lewis (1954) conditions of capital and skill shortage could keep down c_v, while c_f can be kept down by policy to make development finance available at rates that are low enough to encourage growth of investment in capital production. At the same time, institutional development policy to lower c_f and hence c_{tot} and stimulate an increase in production of intermediate and final capital will eventually lower $F'(x)$, given p, and will therefore increase $g - F'(x)$ and hence the net rate of reinvestment of the surplus generated by the assets used to employ x in production.

Again here, equation (57) has no term in $\frac{dx}{dt}$ and its solution cannot be derived from the equation by any choice of the arbitrary constants of integration in its integral. Yet, though without the ability to appeal to uniqueness and existence theorems to determine if there is an equilibrium solution, we can still find an optimal solution $x = x^*_{tot}$ which is a constant associated with the total cost c_{tot} that satisfies the equation. Then, as before, from the state equation in (30), and for $\phi = 1$, we will also have the optimal amount of knowledge, skills and self-confidence employed in production and well as the optimal stock of capital as $K_{tot} = \frac{F(x^*_{tot})}{x^*_{tot}}$.

The derivation gives a general equilibrium solution of the financing capacity maximization problem as: (i) $x = x^*_{tot}$; (ii) $K_{tot} = \frac{F(x^*_{tot})}{x^*_{tot}}$; (iii) $\phi = 1$; and (iv) $S_{tot} = \gamma K_{tot}$, with $x^*_{tot} > x^*_{var}$ since $c_{tot} > c_{var}$. Together with equation (57), these solution values characterise aggregate competitive market behaviour in equilibrium as the market participants pursue the maximum profit-based financing capacity defined in equation (20). This is determined in terms of the optimal level of savings as:

$$58.\ S_{tot} = \frac{\gamma}{\phi}\frac{px^*_{tot} - c_{tot}}{c_{tot}}(g - F'(x^*_{tot}))$$

If it is assumed that c_{tot} is set by conditions in the factor markets and that γ is given, then the optimal saving rate is determined by the optimal competitive strategy to attain the optimal values of p, ϕ and x^*_{tot} in equation (58).

5. A View of Competitive Strategy

Given any set of initial conditions, the optimal competitive strategy can be described by the requirements for attaining it as fast as possible, in a way that reflects what actual market participants do.

Competitive Strategy and the Attainment of $\phi = 1$

One critical element of the optimal solution is $\phi = \frac{E_n N}{K} = 1$. If the initial condition is that $\phi < 1$, it means that the level of knowledge, skills and worker effort ($E_n N$) is too low to efficiently operate the physical capital, and generate the net cash flows required to bring the acquisition and utilization of x up to the optimum x^*_{tot} and support the rate of reinvestment required to bring the capital stock up to $K_{tot} = \frac{F(x^*_{tot})}{x^*_{tot}}$. This situation tends to arise under conditions in which international market and institutional processes force adoption of a stock of K that embodies critical knowledge not matched by the level of knowledge, skills, and self-confidence of workers ($E_n N$). The situation also tends to arise under Lewis (1954) conditions when the supply of capital is too small to employ all available workers and there are large numbers of available workers with low levels of knowledge, skills, and self-confidence. The consequence is usually inefficient application of K in the production process, and therefore variable and fixed costs per unit of x that are relatively high when compared with international competitors, while ϕKx is relatively low. At any given price, the effect is that the returns defined in equation (20) are relatively low. Also, the marginal values (the rate of growth of the present value of surplus due to growth of x and K at t), defined by $\mu(t)$ and $\lambda(t)$ in equations (48) and (53) respectively, are also relatively low. Thus, to grow the marginal returns from growth of x and K to the optimum, the competitor must identify the fastest path from $\phi < 1$ to $\phi = 1$, though not in isolation.

To address the need for a stock of capital and an adequate level of knowledge, skills and self-confidence to fully employ the labour force while bringing ϕ up to its optimal level, it would be necessary to encourage growth of both $E_n N$ and K, through production where possible. Growth of K to a new optimum requires growth of $\frac{F(x^*_{tot})}{x^*_{tot}}$ and supporting increase of S, backed by policies to constrain or reduce c_v and c_f while removing barriers to increase of $F'(x)$.

The design of competitive strategy to bring ϕ up to its optimal level in this context must focus on how $E_n N$ grows as K grows. Consider ϕ in its dynamic form stimulated by continuous technological and institutional progress. That is:

59. $$\phi = \frac{N\frac{dE_n}{dK} + E_n\frac{dN}{dK}}{(1+\frac{Kd\phi}{\phi dK})}$$

The definition is dynamic in the sense that it expresses the effect on the deployed knowledge, skills and self-confidence of workers of increases in the amount of capital owned by capitalists, which embeds information that allows their application and operation without an individual or firm having detailed knowledge of their inner workings.

If $\phi < 1$, then it must grow to achieve the maximum of 1. In that case, $N\frac{dE_n}{dK} + E_n\frac{dN}{dK}$ must grow faster than $(1 + \frac{Kd\phi}{\phi dK})$ while $\frac{d\phi}{\phi dt}$ is growing. The required pattern of growth follows from the fact that:

60. $$\frac{d\phi}{\phi dt} = \frac{d(E_n N)}{E_n N dt} - \frac{dK}{K dt}$$

Equation (60) can then be read to indicate that for ϕ to grow to 1 from an initial condition of $\phi < 1$, the optimal competitive strategy is to grow the knowledge, skills and self-confidence of workers and managers as fast as possible relative to the rate of growth of the stock of complementary (capitalist-owned) capital put to work to produce value with x. The nature, quality and reliability of inputs and output also adjust with such changes in the knowledge, skills and self-confidence of workers and managers as well as the changes in the stock of capitalist-owned assets. Then, once ϕ attains the maximum value of 1, $\frac{d\phi}{\phi dt} = 0$ and the optimum is maintained when the competitor grows the knowledge, skills and self-confidence of workers and managers at the same rate as the stock of capital. Here, the institutional change driving $\frac{dK}{Kdt}$ would normally take the form of introduction of inclusive and innovative development banking institutions that make targeted credit at lowered c_f available to support capital production. Simultaneously, the institutional change driving $\frac{d(E_n N)}{E_n N dt}$ would require access to similar targeted credit facilities complemented by reforms to improve the underlying academic schooling, skill-intensive schooling, and tertiary skills training systems.

However, this adjustment process is continually disrupted by technological shifts that make ϕ piecewise continuous, and with it all the elements of the general solution derived above. Any function f is piecewise continuous on a closed interval $[a, b]$ if it is continuous at all points on the interval except for a finite number of points at each of which f has both a finite left-hand limit and a finite right-hand limit. This property of piecewise continuity is imposed by continuous shifts and changes of the totality of current scientific and experimental knowledge, skills, methods and processes, machines, tools, and infrastructure that can be produced and used with natural and created intermediate inputs to produce or consume goods and services.

Markets in the Caribbean face the condition that when the machines, tools, and infrastructure shift because of technological change in the global economy, so do the required knowledge, skills, and self-confidence of workers, leading to shifts of ϕ, and hence of x^*_{tot}; K_{tot}; and S_{tot}. Since these elements are all piecewise continuous, we could look at competitive strategy as subject to shifts of technology. Technological change tends to create a finite institution-time horizon (t_1) over which current scientific and experimental knowledge and skills, methods and process, and related machines, tools and infrastructure will be relevant. From any given set, $\{\phi_0 < 1; x_0 < x^*_{tot}; K_0 < K_{tot}; S_0 < S_{tot}\}$, market participants must achieve the optimal levels $\phi = 1$, and hence x^*_{tot}; K_{tot}; and S_{tot} before t_1. This is what happens when technical progress is of concern to firms and industries. Technical change will cause $\phi < 1$ and they will have some limited time $[0, t_1]$ over which to achieve some optimal result from existing technique. Moreover, since there will be a repeat of this problem after every round of technological progress, update of machinery, tools, and infrastructure, and change of technique. So, competitors tend to look at competitive strategy in terms of following ϕ; x^*_{tot}; K_{tot}; and S_{tot} up to t_1, then switching to some other strategy that exhausts all or most of the available x and K, and then building them up again, this time with a new technique. Furthermore, with technical progress, ϕ, x^*_{tot}, K_{tot}, and S_{tot} are not constants but rather shift over time. That is, competitors change their strategy over time as they move to some finite time horizon. This view of competitive technical strategy is close to what firms pursuing profits actually do in the light of anticipations of changes of techniques in a dynamic technological environment, and clearly it can be described coherently by the solutions generated with the maximum principle. The institutional development process would have to be very versatile to keep up with and motivate such a dynamic adjustment process.

Competitive Strategy and Attainment of x^*_{tot}

In the Caribbean economy, one option for finding an optimal solution $x = x^*_{tot}$ which is a constant associated with the total cost c_{tot} that satisfies equation (57) is to follow

Lewis (1954) and consider the unconstrained maximization of the rate of profit (r) under surplus labour conditions, as capital K accumulates continuously over time. In this case, considering the total value added by the unit, Y, profit to the capitalist organising production is defined by:

$$61.\ rK = pY - c_v\widetilde{N} - c_f K$$

where it is assumed for convenience that $g + \gamma = 1$ in equation (56).

Since $Y = f(\widetilde{N}(K(t)))$, taking the derivative of (61) with respect to K and collecting terms gives the rate of profit driving savings as:

$$62.\ \ r = \frac{1}{(1+\frac{Kdr}{rdK})}\left[p\frac{Y}{\widetilde{N}}\left(\frac{\widetilde{N}dY}{Yd\widetilde{N}} + \frac{\widetilde{N}dp}{pd\widetilde{N}}\right) - c_v\left(1 + \frac{\widetilde{N}dc_v}{c_v d\widetilde{N}}\right) - (c_f\frac{dK}{d\widetilde{N}} + \frac{1}{\phi}\frac{\widetilde{N}dc_f}{c_f d\widetilde{N}})\right]\frac{d\widetilde{N}}{dK}$$

where, $\frac{Y}{\widetilde{N}}$ is the average product of the knowledge, skills and self-confidence of workers, and $\frac{\widetilde{N}dY}{Yd\widetilde{N}}$ and $\frac{\widetilde{N}dp}{pd\widetilde{N}}$ are respectively the elasticities of output and price with respect to knowledge, skills and self-confidence. Since $\phi = \frac{\widetilde{N}}{K}$, it follows that:

$$63.\ \frac{dK}{d\widetilde{N}} = \frac{1}{\phi(1+\frac{Kd\phi}{\phi dK})}$$

Using equation (63) in equation (62) gives:

$$64.\ \ r = \frac{1}{(1+\frac{Kdr}{rdK})}\left[p\frac{Y}{\widetilde{N}}\left(\frac{\widetilde{N}dY}{Yd\widetilde{N}} + \frac{\widetilde{N}dp}{pd\widetilde{N}}\right) - c_v\left(1 + \frac{\widetilde{N}dc_v}{c_v d\widetilde{N}}\right) - (\frac{c_f}{\phi(1+\frac{Kd\phi}{\phi dK})} + \frac{1}{\phi}\frac{\widetilde{N}dc_f}{c_f d\widetilde{N}})\right]\frac{d\widetilde{N}}{dK}$$

If $\phi \to 1$, and surplus-labour conditions imply that $\frac{\widetilde{N}dc_v}{c_v d\widetilde{N}} = 0$, while c_f is set exogenously, so that $\frac{\widetilde{N}dc_f}{c_f d\widetilde{N}} = 0$, then equation (64) reduces to:

$$65.\ \ r = \frac{1}{(1+\frac{Kdr}{rdK})}\left[p\frac{Y}{\widetilde{N}}\left(\frac{\widetilde{N}dY}{Yd\widetilde{N}} + \frac{\widetilde{N}dp}{pd\widetilde{N}}\right) - (c_v + c_f)\right]$$

Thus, the rate of profit is maximized when $\frac{Y}{\widetilde{N}}\left(\frac{\widetilde{N}dY}{Yd\widetilde{N}} + \frac{\widetilde{N}dp}{pd\widetilde{N}}\right)$ takes its maximum value. Thus, given c_{tot} and $\phi = 1$ in equation (58), this maximum value is the Lewis candidate for the constant x_{tot} that satisfies equation (57) and maximizes profits. That is:

$$66.\ x_{tot} = \frac{Y}{\widetilde{N}}\left(\frac{\widetilde{N}dY}{Yd\widetilde{N}} + \frac{\widetilde{N}dp}{pd\widetilde{N}}\right)_{max}$$

Here too, the elements defining x_{tot}, that is $\frac{Y}{\widetilde{N}}$, $\frac{\widetilde{N}dY}{Yd\widetilde{N}}$ and $\frac{\widetilde{N}dp}{pd\widetilde{N}}$ are subject to shifts over time, depending on how institutional progress allows adjustment of $\widetilde{N}$ in the production process. Thus, as with the attainment of $\phi = 1$, attainment of the maximum S_{opt} in equation (58) through x_{tot} also depends on how institutional progress allows growth of $\widetilde{N}$ in the production process.

Adjustment could emphasize growth of the average level of knowledge, skills and self-confidence or growth of the number of workers employed. This follows from the fact that $\frac{d\widetilde{N}}{dt} = \left(N\frac{dE_n}{dK} + E_n\frac{dN}{dK}\right)\frac{dK}{dt}$. When system-wide institutional progress is rapid and dt represents a short time schedule, then organisational adjustment can emphasize $\frac{dE_n}{dt}$ at a given level of employment N, and the effect is to grow $\frac{dY}{d\widetilde{N}}$ and $\frac{Y}{\widetilde{N}}$ while generating increasing returns. On the other hand, if institutional progress is slow and dt represents a long time schedule, adjustment must emphasize $\frac{dN}{dt}$ over growth of the average level of knowledge and skills, dE_n. Then $\frac{dY}{d\tilde{N}}$ and $\frac{Y}{\tilde{N}}$ will tend to fall. The result is that, in any interval $[0, t_1]$ the maximum ($\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$) that is attained depends on the extent to which institutional progress is engineered to allow adjustment of $\tilde{N}$ by emphasizing growth of the average level of knowledge, skills and self-confidence of workers as the stock of capital accumulates, i.e., $\frac{dE_n}{dK}$.

Competitive Strategy and Determination of p

An important element of competitive strategy is determination of p in equation (58). For this, there is no general principle, since pricing options vary with the type of industry. In the Caribbean case, there are two broad types of sectors with different options for pricing strategy.

One is the sector that specialises in production of natural-resource-based intermediate manufactured goods for export or production of tourism services. It is dominated by foreign capital and fully integrated into the global inter-industry trading system. In this case, p is an exogenous variable, determined in the international marketplace. It is subject to random positive or negative shocks that create swings in profits in equation (58) similar to those identified by Best (1968).

The other is the sector that produces domestic capital services. This sector is dominated by domestic capital and produces output for supply of domestic final demand as well as for participation in the global intra-industry trading system. In this case, the industry can set p strategically to reflect applicable price, advertising, and other elasticities of demand for its output as well as the market power achieved by the capacity to produce capital, adjust ϕ and introduce winning solutions to problems thrown up by the market. The extent of market power is captured by the term $\frac{\tilde{N}dp}{pd\tilde{N}}$ that affects x_{tot} in equation (66), but this can reasonably be treated as finite. As with the attainment of $\phi = 1$, its extent and the related optimal value of p depends on how institutional progress influences the adjustment of the knowledge, skills and self-confidence of workers in the industry and hence the adjustment of the nature, quality and reliability of inputs and outputs. In general, institutional progress is modest at best.

6. The Optimal Scale of Financing Capacity and the Stability Problem

The foregoing results can shed some light on the optimal scale and stability of the financing capacity of the competing unit as measured by the scale of savings generated by the capitalist. First, the foregoing results imply that the applicable form of equation (58) is:

$$67. \quad S_{tot} = \frac{\gamma}{\phi} \frac{p\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max} - c_{tot}}{c_{tot}} (g - F'(x^*_{tot}))$$

Second, regarding the stability of equation (67), we know that as capital accumulates and technical progress occurs, $\phi \rightarrow 1$ on a piecewise continuous basis. So, piecewise continuous technical progress does not induce indefinite growth of S_{tot} since the influence of ϕ on S_{tot} falls to zero as $\phi \rightarrow 1$. Then, since x_{tot}^* in equation (66) is a constant subject to shifts over time as institutions and hence technology develop, so would be S_{tot}. However, even with repeated shifts over time, S_{tot} would tend to be associated with a finite sequence of equilibria once $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$ is finite. Next, the average product of labour in subsistence activity would set a floor to c_{var}. As observed by Lewis (1954), this floor would tend to rise as $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)$ approaches $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$, subsistence activity is marginalized and the economy approaches full employment in the capitalist sector. The increase would tend to create convergence between $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$ and c_{var}.

Savings are normally held in the financial sector, so it also holds that c_f will tend to fall in the financial markets as $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$ is approached and savings become increasingly abundant. However, here, policymaking institutions normally intervene to ensure stability in the financial markets. They set a floor below c_f and a ceiling above it. As, c_f falls with growth of $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)$, it offsets the rise in c_{var} and the overall effect is to prevent convergence of c_{tot} and $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$ enabling continuous growth of S_{tot} even in a context of repeated improvements in the marginal and average products of the knowledge, skills, and self-confidence of workers that cause $c_{var} \rightarrow \frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$.

Further, increased investment and institutional progress also cause $F'(x_{tot}^*)$ to grow. This serves as an environmental conservation mechanism since the general effect of $(g - F'(x_{tot}^*))$ would be to lower the rate of reinvestment of profits and prevent indefinite growth of S_{tot} under conditions when c_{tot} is falling. Finally, under conditions in which $\frac{\tilde{N}dp}{pd\tilde{N}}$ tends to rise rapidly, such as can result when the monetary tap is opened to accelerate the product of capital and the economy approaches full employment, the policymakers also tend to intervene to raise c_f to complement the increase in c_{var} and cause relatively rapid convergence of c_{tot} and $\frac{Y}{\tilde{N}}\left(\frac{\tilde{N}dY}{Yd\tilde{N}} + \frac{\tilde{N}dp}{pd\tilde{N}}\right)_{max}$. The joint effect of this policy-induced convergence and the environmental conservation mechanism tends to prevent indefinite growth of S_{tot}, avoiding the instability of the savings process as the economy approaches full employment.

7. Summary Results

The Lewis (1954) unconstrained profit-creation model proposed that stability in the creation of savings as an economy develops can be achieved because in the process the subsistence sector becomes increasingly marginalised, its average product of labour rises, and the labour market is pushed to full employment in the capitalist sectors. This would raise the wage rate in the capitalist sector and cause its convergence to the marginal product of labour even as the latter increases with investment. However, Sraffa's (1926, 1930) insightful critique of the standard theory of the firm also implies the possibility of solution instability, in the sense of indefinite growth of financing capacity in the Lewis process. Continuous creation of technology-driven increasing returns, economies of scope, and learning by doing as a response to changing labour market conditions, would cause the growth of the marginal product of labour to outpace the growth of the wage rate and lead to indefinite growth of profits and savings that would perpetually lower the cost of capital. Any rising wages in the labour market could be sufficiently offset by the falling cost of capital, producing either a stationary or falling unit cost production in combination with a rising marginal product of labour that cause instability in the Lewis process.

This paper resolved the instability problem by specifying a profit-creation process that is constrained by the supply of the intermediate inputs into production and by the process of supplying savings to validate future investment. The net intermediate resource availability constraint is defined in terms of the applicable growth of supply and the demand for its use, which reflects the relevant production function technique. The net savings constraint is defined in terms of gross savings and the provisions for depreciation due to the use and obsolescence of the stock of capital. Other non-negativity initial and boundary constraints on technology and savings also apply. Maximization of accumulated financing capacity over the planning horizon is assumed.

The solution of the constrained profit-maximization problem is obtained by applying optimal control theory and the Pontryagin maximum principle. The optimal solution is defined in terms of an optimal value of the intermediate capital used, the optimal savings rate, and the optimal technique of production. The optimal savings rate is found to depend on price, the optimal technology, the optimal value of the intermediate capital used, and the unit cost of production. The optimal value of the intermediate capital used is shown to be appropriately chosen as the Lewisian average product of the knowledge, skills and self-confidence of workers augmented by the elasticities of output and price with respect to that knowledge, skill, and self-confidence. Of importance, the elasticity of output with respect to the knowledge, skills and self-confidence of workers depends on the marginal product of labour.

The optimal technology has a value of 1, which is to say a 1:1 ratio of knowledge, skills and self-confidence of workers, and the capital contributed by capitalists. It is attained through a piecewise-continuous adjustment process as the competing unit responds to changes in the stock of capital generated in the foreign economy. The typical initial condition is that the production technique is less than 1. This implies that, on a piecewise basis, in each period of technical change to the stock of capital, the rate of growth of the knowledge, skills, and self-confidence of workers must be as high as possible above the rate of growth of the capital stock.

An important element of competitive strategy is determination of the price and unit costs that influences the savings rate. Unit costs can be treated as exogenously determined by conditions in the market for labour and by public policy on the cost of finance. No general principle is attainable for the relevant price, since pricing options vary with the type of industry considered.

In the set of industries that specialises in production of natural-resource-based intermediate manufactured goods for export or production of tourism services, price is set exogenously in the international market, and is subject to random positive or negative shocks that create swings in profits similar to those identified by Best (1968). In the set of industries that produces domestic capital services, price can be set strategically to reflect applicable elasticities of demand for its output as well as the market power created by the capacity to produce capital and introduce winning solutions to problems thrown up by the market. The extent of market power is captured by the elasticity of price with respect to the knowledge, skills, and self-confidence of workers.

In every case, the approach to these optima depends on how institutional progress enables adjustment of the knowledge, skills, and self-confidence of workers. In particular, rapid institutional progress enables a high degree of reliance on the growth of the average level of knowledge, skills, and self-confidence of workers, leading to related improvements in the type, quality and reliability of inputs and outputs.

Finally, with respect to the optimal scale of financing capacity at any time, piecewise continuous technical progress does not induce its indefinite growth. Even with repeated shifts over time, the pool of savings at any time would tend to a finite sequence of equilibria as long as the optimal level of use of intermediate inputs is finite. As the process of development progresses, and the average product of labour in subsistence activity rises in conjunction with the marginalisation of the subsistence sector, the wage rate converges to the maximum level of use of intermediate inputs. The process also drives down the average cost of savings, offsetting the effects of the rising wage and enabling the unstable accumulation of savings even in the context of a policy floor. However, as the economy develops the operational environmental conservation mechanism also tends to lower the rate of reinvestment of profits and prevent indefinite growth of the savings owned by capitalists. When the financing of capital production induces inflation along with the rising wage rate, the institutional policymakers induce a rising cost of capital that causes convergence of the augmented average product of labour and the average cost of production and brings the growth of savings to a halt. Thus, two complementary mechanisms exist that would prevent indefinite growth the financing capacity and induce long run stability of the development process. One is policy-induced increases in the cost of finance to complement the rising wage rate and restrain the inflation caused by investment financing for development. The other is the operational environmental conservation mechanism which tends to lower the rate of reinvestment of profits.

References

Best, L. A. (1980). ***International Co-operation*** in the ***Industrialization*** **Process**: The Case of *Trinidad* and Tobago. Port of Spain: TTRIWI. Published in "Industry 2000, New Perspectives": Collected background Papers, Vol.6, New York: UNIDO, pp. 153-299.

Best, L.A and Polanyi-Levitt, K. (1969). **Externally Propelled Growth and Industrialization in the Caribbean**, 4 Volumes, Mimeograph, Centre for Developing Areas Studies, Montreal: McGill University.

Clark, C. W. (1976). *Mathematical Bioeconomics*: The Optimal Management of Natural Resources. New York: John Wiley & Sons.

Clark, C.W. (2010). *Mathematical Bioeconomics*: The Mathematics of Conservation. New York: John Wiley & Sons.

James, V. and Hamilton, R. (2022). Strategic Factors in Economic Development Revisited. ***Development Essays***, Vol. 1. No. 1.

Lewis, W. A. (1950). The Industrialisation of the British West Indies, *Caribbean Economic Review,* (May).

Lewis, W. A. (1954). Economic Development with Unlimited Supplies of Labour. *Manchester School of Economics and Social Studies,* 22: 417-419.

Polanyi, K. (1944). ***Origins of Our Time:*** The Great Transformation. New York: Farrar & Rinehart.

Polanyi, K., Arensburg, C.M., and Pearson, H.W. (eds) (1957). ***Trade and Markets in the Early Empires.*** Glencoe, Ill.: Free Press and Falcon's Wing Press.

Pontryagin, L.S., Boltyanskii, V.S., Gamkrelidze, R.V., and Mishchenko, E.F. (1962). *The Mathematical Theory of Optimal Processes.* New York: Wiley-Interscience.

Smith, A. (1776). ***The Wealth of Nations****: Books I – III*, London: Penguin Classics, 1999.

Sraffa, P. (1926). The Laws of Returns Under Competitive Conditions. *The Economic Journal,* 36, December: 535-550.

Sraffa, P. (1930). A Criticism. In "Increasing Returns and the Representative Firm, A Symposium", *The Economic Journal,* 40, March: 79-89.